Primljen / Received: 22.2.2023.
Ispravljen / Corrected: 13.5.2023.
Prihvaćen / Accepted: 6.10.2023.
Dostupno online / Available online: 10.12.2023.

Approximate calculation of circular reinforced concrete columns with nomograms

Authors:

Li Lu, MCE
Kunming University of Science and Technology
Departement od Civil Engineering
China
1228982624@qq.com

Prof. **Zhou Dong-hua**, PhD. CE
Kunming University of Science and Technology
Departement od Civil Engineering
China
806029639@qq.com
Corresponding author

Assit.Prof. **Shuang Chao**, PhD. CE
Kunming University of Science and Technology
Departement od Civil Engineering
China
1403094948 @qq.com

Research Paper

Li Lu, Zhou Dong-hua, Shuang Chao

Approximate calculation of circular reinforced concrete columns with nomograms

Calculating the reinforcement of circular reinforced concrete (RC) columns involves not only the dual nonlinearity of the geometry and material but also the nonlinearity of the section width. Accurate solutions require iterative calculations. To develop the calculation method manually, the model column method was proposed to compute the second-order effect of the columns, and the strain method was used to calculate the ultimate strength of the sections analytically. The nomograms required to calculate the reinforcing steel content of the columns without iterations were obtained. The nomogram for calculating the section bearing capacity and reinforcing steel has three parameters (axial force, bending moment, and mechanical ratio of the reinforcing steel). Further, the nomogram for calculating the column bearing capacity and reinforcing steel has five parameters (axial force, bending moment, curvature, slenderness ratio, and mechanical ratio of the reinforcing steel), and the relationship between the five parameters can be expressed in a plan, which makes the application convenient. Finally, the calculation results of the nomograph were compared with those of the existing approximate calculation formulas and exact numerical methods, and the accuracy of the nomograms was verified.

Key words:
circular RC column, second-order effect, compression and bending members, reinforcement calculation, nomogram

Prethodno priopćenje

Li Lu, Zhou Dong-hua, Shuang Chao

Približni proračun kružnih AB stupova uz pomoć nomograma

Proračun armature kružnih AB stupova uključuje ne samo dualnu nelinearnost geometrije i materijala, već i nelinearnost širine presjeka. Precizna rješenja zahtijevaju iterativne proračune. Da bi se ručno izradila metoda proračuna, predložena je metoda modela stupa kako bi se izračunao učinak drugog reda stupova, a metoda deformacije primijenjena je kako bi se analitički izračunala granična nosivost presjeka. Dobiveni su nomogrami potrebni za proračun količine armature u stupovima bez iteracije. Nomogram za proračun kapaciteta nosivosti presjeka i količine armature ima tri parametra (uzdužnu silu, moment savijanja i mehanički koeficijent armiranja). Nadalje, nomogram za proračun kapaciteta nosivosti stupa i količine armature ima pet parametara (uzdužnu silu, moment savijanja, zakrivljenost, koeficijent vitkosti i mehanički koeficijent armiranja), a odnos između tih pet parametara može se izraziti nacrtom zbog čega je ta primjena prikladna. Naposljetku, rezultati proračuna koje prikazuje nomograf uspoređeni su s onima koji su dobiveni postojećim formulama za približan proračun te s onima koji su dobiveni točnim numeričkim metodama, pa je tako potvrđena točnost nomograma.

Ključne riječi:
kružni AB stup, učinak drugog reda, elementi izloženi tlaku i savijanju, proračun armature, nomogram

1. Introduction

To compute the resistance of circular RC sections, a widespread practical approach for analyzing and designing sections is based on using an interaction diagram [1]. The circular RC section resistance calculation using the simplified formulae in EC2 requires iterations [2]. Interaction diagrams and formulas are constructed using computer-aided numerical programs [3]. A closed-form compact formulation does not require a numerical solution of the proposed equilibrium equation [4]. Without considering the second-order effect, calculating the strength of a circular RC section becomes extremely complicated. The second-order effect of columns subjected to a combined bending moment and axial force cannot be ignored because the slenderness ratio increases and bearing capacity of the columns decreases [5]. Therefore, two methods based on the nominal stiffness and curvature were suggested [2, 6]. Moreover, several methods have been proposed for calculating the ultimate resistance and ductility of rectangular RC columns [5, 7-16], and a simplified analytical relationship between the bending moment and curvature of circular RC cross sections has been proposed [17, 18].
This study aimed to provide calculations for circular RC columns without computer iterative and numerical calculations. The strain method adopted in this study [19], that is, the stress and resistance, can be calculated according to the known strain and constitutive relationship of concrete and reinforcing steel without any simplified or iterative calculations. In addition, the exact relationship between the bending moment and curvature can be computed, and the deflection and ductility of the columns can be determined from the strain of the cross-sections. Hence, it calculates the reinforcing steel and second-order bending moment of circular RC columns manually, possibly because the internal cross-sectional resistance is associated with the external load action. A comparative analysis was performed using approximate formulas [4] for the resistance of the sections. Using concrete examples, the resistance calculation method (nomogram) for circular RC columns considering the second-order effect was compared with the exact numerical method, and the accuracy of the nomogram was verified. The proposed nomogram effectively determined the initial section of the columns at the preliminary design stage and verified the section at the verification stage.
In this study, the first-order calculation of the sections exhibits some similarities with the calculation methods in references [19-21], and the second-order calculation of the columns exhibits some similarities with [22].

2. Calculation of second-order deflection and second-order bending moment of columns

2.1. Model column that considers the second-order effect

As shown in the method based on the nominal stiffness in EC2, the key to calculating the deformation is to determine the change in the section stiffness (EI). The section parameters, such as depth, width, and reinforcing steel, must be known to calculate the section stiffness, and the reinforcing steel of the section is an unknown quantity. Therefore, the key to calculating the section stiffness is to determine the distribution and magnitude of the curvature. Therefore, a cantilever column was selected as the representative model column for the calculations (model column shown in Figure 1).

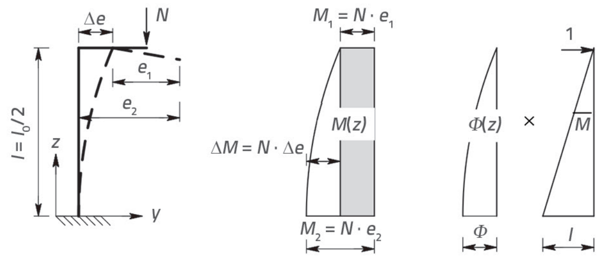

Figure 1. Column calculation diagram (model column: a cantilever column)

As shown in Figure 1, M_2 is the total bending moment at the fixed end of the column, M_1 is the first-order bending moment, ΔM is the second-order bending moment, and e_1, Δe and e_2 refer to the first-order, second-order, and total eccentricities, respectively.
Assuming that the curvature distribution curve along the length of the column is a parabola and that the curvature of the section at the fixed end of the column is Φ, the calculation formula is shown in Eq. (4). Using the bending moment area method, Δe can be computed as follows:

$$\Delta e = \int \Phi(z)\overline{M}dz = \frac{2}{3}\cdot l\cdot \Phi \cdot \frac{5}{8}\cdot l = \frac{5}{48}\Phi l_0^2 \qquad (1)$$

Then, M_2 at the fixed end of the column can be computed as follows:

$$M_2 = Ne_1 + N\Delta e = Ne_1 + \frac{5}{48}N\Phi l_0^2 \qquad (2)$$

The actual length of the cantilever column is replaced by the effective length l_0, e.g. ($l_0 = 2l$), because any column can be isolated from structures [23], and it becomes an independent single column base on the equal effective length l_0. Eq. (2) can be used to calculate the second-order deflection and second-order bending moment caused by the P-Δ (sway frame) and P-Δ (non-sway frame) effects of any column in the structure and the magnitude of the P-Δ and P-δ effects is therefore determined by the effective length l_0.
The total bending moment M_2 can be obtained using Eq. (2), and the dimensionless bending moment m_2 is computed as follows:

$$m_2 = \frac{M_2}{\pi r^2 d f_{cd}} = \underbrace{m_1}_{\text{first-order}} + \underbrace{\Delta m}_{\text{second-order}} = \underbrace{m_1}_{\text{first-order}} + \underbrace{\frac{5}{48}n\left(\frac{l_0}{d}\right)^2\phi}_{\text{second-order}} \qquad (3)$$

In Eq. (3): $m_2 = m_1 + \Delta m$, $\phi = \Phi d$. Eq. (3) is a linear equation with the dimensionless curvature ϕ as the independent variable; its intercept is m_1 (dimensionless first-order bending moment), and its slope is -0,104 · n · $(l_0/d)^2$, as shown in Figure 2. If an additional coordinate axis of $\sqrt{-n} \cdot l_0/d$ is added to the right side of Fig. 2, the slope can be directly determined using $\sqrt{-n} \cdot l_0/d$. Figure 2 shows part of the reinforcing steel calculation diagram.

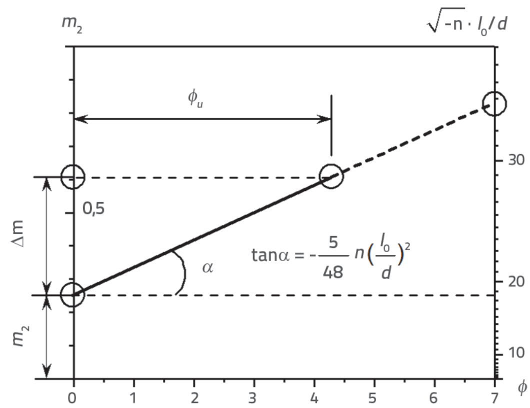

Figure 2. Second-order eccentric distance of external effect

As shown in Figure 2, if m_1 and $\sqrt{-n} \cdot l_0/d$ are known, an oblique straight line can be determined. The dimensionless second-order bending moment Δm equals to the increment of the oblique line, which increases with the dimensionless curvature ϕ. However, ϕ cannot exceed the dimensionless ultimate curvature ϕ_u and ϕ_u varies with M_2, N and reinforcing steel of the section. In summary, Δm is controlled by ϕ_u of the section.

3. Calculation of ultimate curvature and resistance of sections

3.1. Strain distributions of concrete and reinforcing steel

Based on the geometric relationship between the strain shown in Figure 3 and plane hypothesis, the section curvature Φ can be computed as follows:

$$\Phi = \frac{\varepsilon_{c1} - \varepsilon_{c2}}{d} \tag{4}$$

After making Φ in Eq. (4) to be dimensionless, we obtain:

$$\phi = \Phi \cdot d = \varepsilon_{c1} - \varepsilon_{c2} \tag{5}$$

where ε_{c1}, ε_{c2} and d are the upper edge strain, lower edge strain, and diameter of the sections, respectively, as shown in Figure 3.

The section resistance and dimensionless ultimate curvature ϕ_u of the sections are computed using the known section ultimate strains. According to EC2, the ultimate compressive strain of concrete and tensile strain of reinforcing steel is -3.5 ‰ and 20 ‰, respectively. Therefore, all the strain distributions in the section can be determined and divided into five zones. In each zone, at least one fiber in the section depth reached the ultimate compressive strain of the concrete or the ultimate tensile strain of the reinforcing steel (Figure 3).

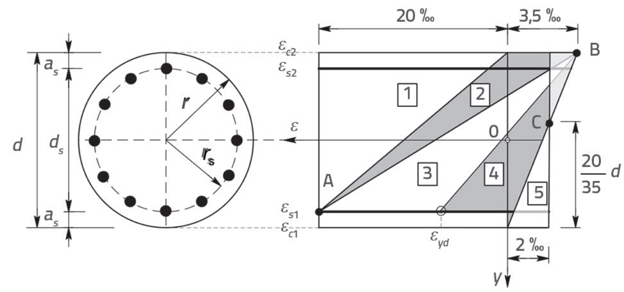

Figure 3. Circular reinforced concrete cross-section: geometry and ultimate strain distributions

Zone 1 (Rotation around point A)
The neutral axis falls from infinity above the section to the upper edge of the section. The stress state transitions from uniform tension to tension with small eccentricity. At the left boundary of the region, the strain distribution represents uniform tension, at which the reinforcing steel strain on both edges of the section reaches the value of ε_s = 20 ‰.

Zone 2 (rotation around point A) and **Zones 3 and 4** (rotation around point B)
Part of the section was under compression, and the neutral axis moved gradually from the upper edge to the lower edge of the section. The load cases transitioned from tension with low eccentricity to tension with high eccentricity, pure bending, compression with high eccentricity, and compression with low eccentricity.

Zone 5 (rotation around Point C)
This is a swept region where the right boundary line of region 4 is rotated counterclockwise around point C to a vertical position (point C can be calculated from the geometric relationship between the top- and bottom-edge strains). The entire section was under compression, and the neutral axis moved from the bottom of the section to infinity below the section. The load cases in the region may be compressed with a small eccentricity and uniform compression.

The strain changed continuously from zone 1 to zone 5, and the corresponding load cases also gradually transitioned.

3.2. Constitutive relationship

According to the constitutive relationship of RC in EC 2, the parabola-rectangular strain–stress relationship was adopted for the compression concrete, and the elastic–perfectly plastic strain–stress relationship was applied to the reinforcing steel (Figure 4). This study focused on concrete with a cylindrical characteristic strength f_{ck} less than or equal to 50 MPa, and the tensile strength of the concrete was neglected.

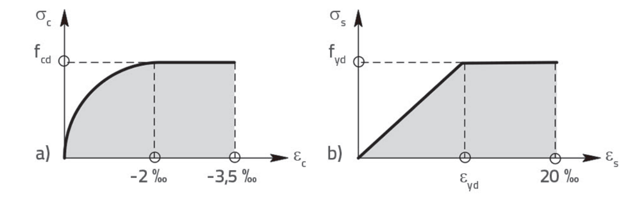

Figure 4. Constitutive relation of materials: a) concrete; b) reinforcing steel

The mathematical expressions for the constitutive relationship between reinforcing steel and concrete in EC 2 are as follows:

$$\sigma_s = \begin{cases} E_s\varepsilon_s & 0 < |\varepsilon_s| < \varepsilon_{yd} \\ f_{yd} & \varepsilon_s \geq \varepsilon_{yd} \\ -f_{yd} & \varepsilon_s \leq -\varepsilon_{yd} \end{cases} \tag{6}$$

$$\sigma_c = \begin{cases} -f_{cd}(1-(1-\dfrac{\varepsilon_c}{-2.0\ ‰})^2) & 0 > \varepsilon_c > -2.0\ ‰ \\ -f_{cd} & -2.0\ ‰ \geq \varepsilon_c \geq -3.5\ ‰ \end{cases} \tag{7}$$

In Eq. (6) and Eq. (7), ε_c, σ_c, f_{cd}, ε_s, σ_s, ε_{yd}, f_{yd} and E_s are the compressive strain and stress, design compressive strength of the concrete, strain and stress of the reinforcing steel, design yield strength and yield strain of the reinforcing steel, and elastic modulus of the reinforcing steel, respectively.

3.3. Stress and resistance of concrete

After determining the ultimate strain distribution of the section, the section resistance was calculated using the known strain.

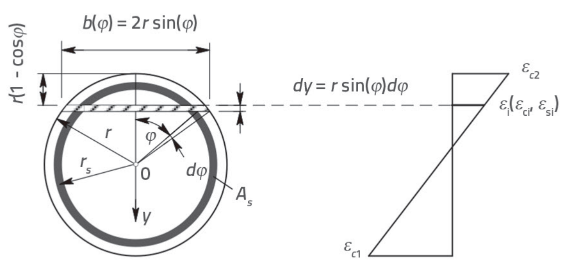

Figure 5. Circular reinforced concrete cross-section: geometry and strains

The strain at any depth within the cross-section can be calculated as:

$$\varepsilon_{ci} = \varepsilon_{c2} - \frac{(1-\cos(\varphi))}{2}(\varepsilon_{c2} - \varepsilon_{c1}) \tag{8}$$

Substituting the value of ε_{ci} from Eq. (8) into Eq. (7), the concrete stress σ_{ci} at any fibre within the cross-section can be calculated, and the axial force and bending moment of concrete can be computed by the integration of stress over the cross-section.

$$\begin{aligned} N_c(\varepsilon_{ci}) &= \int_0^{\pi} \sigma_{ci}(\varepsilon_{ci}) b(\varphi) r \sin(\varphi) d\varphi \\ M_c(\varepsilon_{ci}) &= \int_0^{\pi} \sigma_{ci}(\varepsilon_{ci}) b(\varphi) \cdot r^2 \cos(\varphi)\sin(\varphi) d\varphi \end{aligned} \tag{9}$$

Eq. (9) is a function of the strain at any depth within the cross-section (ε_{ci}). To obtain a more general dimensionless expression, the axial compression ratio n_c and dimensionless bending moment m_c of concrete can be computed as follows:

$$\begin{cases} n_c(\varepsilon_{ci}) = \dfrac{N_c(\varepsilon_{ci})}{\pi r^2 f_{cd}} \\ m_c(\varepsilon_{ci}) = \dfrac{M_c(\varepsilon_{ci})}{\pi r^2 d f_{cd}} \end{cases} \tag{10}$$

3.4. Stress and resistance of reinforcing steel

Assuming that the reinforcement is uniformly distributed at a distance r_s from the center of gravity of the concrete section, the reinforcement area per unit radian is

$$\overline{A_s} = \frac{A_s}{2\pi} \tag{11}$$

The strain of the reinforcing steel bar can be easily obtained from the strain relationship shown in Figure 5. The strain of the reinforcing steel at any fiber in the cross section can be calculated as:

$$\varepsilon_{si}(\varphi) = \varepsilon_{c2} - \frac{(r - r_s\cos(\varphi))}{2r}(\varepsilon_{c2} - \varepsilon_{c1}) \tag{12}$$

Substituting the value of ε_{si} (φ) from Eq. (12) into Eq. (6), the stress of the reinforcing steel of σ_{si} (φ) at any fibre of cross-section can be computed, and the axial force and the bending moment of the reinforcing steel can be obtained by integration.

$$\begin{cases} N_s(\overline{A_s}, \varepsilon_{si}) = 2\int_0^{\pi} \sigma_{si}(\varepsilon_{si}) \overline{A_s} r_s d\varphi \\ M_s(\overline{A_s}, \varepsilon_{si}) = 2\int_0^{\pi} \sigma_{si}(\varepsilon_{si}) \overline{A_s} r_s^2 \cos(\varphi) d\varphi \end{cases} \tag{13}$$

Eq. (14) for dimensionless proceedings is established as follows. The compression ratio n_s and dimensionless bending moment m_s of the reinforcing steel can be obtained as:

$$\begin{cases} n_s(\omega, \varepsilon_{si}) = \dfrac{N_s}{\pi r^2 f_{cd}} = \dfrac{\omega}{\pi f_{yd}} \displaystyle\int_0^{\pi} \sigma_{si}(\varepsilon_{si}) d\varphi \\ m_s(\omega, \varepsilon_{si}) = \dfrac{M_s}{\pi r^2 d f_{cd}} = \dfrac{\omega}{\pi f_{yd}} \dfrac{r_s}{r} \displaystyle\int_0^{\pi} \sigma_{si}(\varepsilon_{si}) \cos(\varphi) d\varphi \end{cases} \tag{14}$$

In Eq. (14), ω (the mechanical ratio of the reinforcing steel) is defined as follows:

$$\omega = \frac{A_s}{\pi r^2} \frac{f_{yd}}{f_{cd}} \tag{15}$$

3.5. Resistance of the full section

The dimensionless resistance of the section is the sum of the dimensionless resistances of concrete and reinforcing steel. The strain ε_{ci} and ε_{si} at any depth in the cross-section can be determined using the section strain ε_{c1} and ε_{c2}. The axial compression force ratio n and dimensionless bending moment m of the section can be expressed by ε_{c1}, ε_{c2} and ω:

$$\begin{cases} n = n_c(\varepsilon_{c1}, \varepsilon_{c2}) + n_s(\varepsilon_{c1}, \varepsilon_{c2}, \omega) \\ m = m_c(\varepsilon_{c1}, \varepsilon_{c2}) + m_s(\varepsilon_{c1}, \varepsilon_{c2}, \omega) \end{cases} \tag{16}$$

The two equations in the Eq. (16) are nonlinear, and each equation contains three independent variables: ε_{c1}, ε_{c2} and ω. When calculating the cross-section resistance, the values of ε_{c1} and ε_{c2} can be obtained according to Figure 3.

3.6. Interaction between dimensionless moment, normal force, and reinforcing steel

Figure 6 can be adopted to compute the reinforcing steel of circular RC sections. When ω and n are known, the ultimate strains ε_{c1} and ε_{c2} can be obtained from the first equation of Eq. (16) and ultimate strain distribution of sections shown in Figure 3. The dimensionless ultimate bending moment m_u can be calculated using the second equation of Eq. (16). The corresponding relationship between n, ω, ε_{c1}, ε_{c2} and m_u is obtained. Therefore, considering ω a constant value, n the abscissa, and the dimensionless ultimate bending moment m_u the ordinate, all curves shown in Figure 6 are drawn.

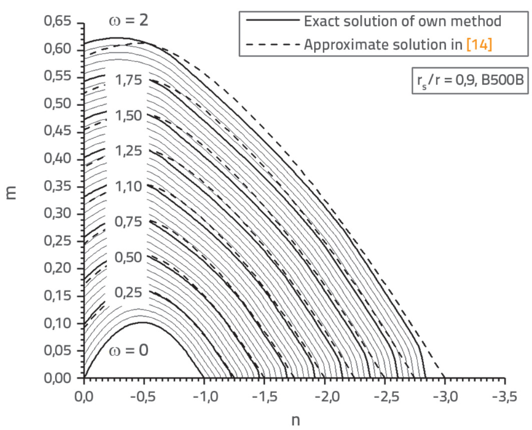

Figure 6. Relationship between the ultimate dimensionless bending-moment and reinforcing steel ratio

Figure 6 shows that the calculation method used in this study is very close to the approximate formula calculation results [4], which verifies the correctness of the calculation method used herein. Figure 6 shows a part of the nomogram adopted to calculate the reinforcing steel of the circular RC columns. The ordinate m in Figure 6 represents the resistance of the sections, which corresponds to the effect of the action shown in Figure 2.

3.7. Interaction between dimensionless normal force, ultimate curvature, and reinforcing steel

When the mechanical ratio of the reinforcing steel ω is known, the ultimate deformation capacity (dimensionless ultimate curvature ϕ_u) varies with axial compression force ratio n. Therefore, the relationship between n and ϕ_u must be considered.
Given the limit strains ε_{c1} and ε_{c2}, in accordance with Figure 3, Eqs. (5) and Eq. (16) can be used to compute ϕ_u, n and m, and analyze their relationships. By setting n as the ordinate and ϕ_u as the abscissa, the curves of the exact relationship between ϕ_u and n can be drawn, as shown in Figure 7. The results calculated using the simplified calculation formulas for EC2 are shown in Figure 7.

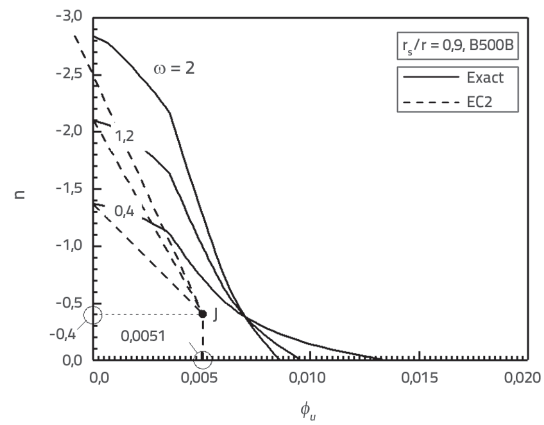

Figure 7. Relationship of axis force ratio and dimensionless ultimate curvature

In Figure 7, the exact relationship between ϕ_u and n is nonlinear; however, the equations in EC2 are linear and significantly different from the exact results.

3.8. Interaction between dimensionless moment, ultimate curvature and normal force

According to the relationship between ϕ_u, n, m_u and ω in Figure 7, the abscissa in Figure 7 is replaced with ϕ_u that calculated using Eq. (5), as shown in Fig. 8.

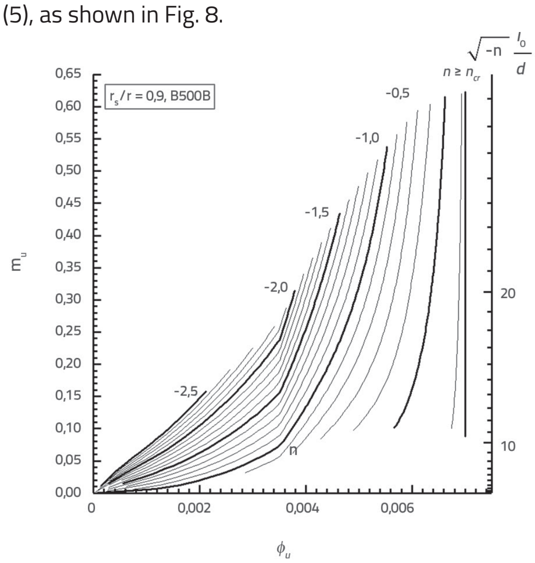

Figure 8. Relationship between the dimensionless total eccentricity and dimensionless ultimate curvature

The following points can be obtained from Figure 8:

- When n is fixed, *m* is increases proportionally by ϕ_u.
- When ω is a constant, such as $\omega = 2$, the larger the n, the smaller the m_u and ϕ_u. This implies the deformation capacity and bending capacity of sections become smaller.

4. Calculation of the mechanical ratio of reinforcing steel considering the second-order effects of columns

4.1. Nomogram for the reinforcing steel calculation of columns

First, Figure 8 and 2 overlap in the plan view because their two axes are the same. Knowing *n*, m_1 and l_0/d, the oblique line in Figure 2 can be used to represent the effect of the action, and the intersection point of the oblique line and the *n* curves in Figure 8 can be located by the given *n*. The ordinate of the intersecting points is the ultimate bending moment of the section m_u, and the abscissa is the dimensionless ultimate curvature ϕ_u of the sections. In addition, the required mechanical ratio ω of the reinforcing steel can be determined using Figure 6 because the figure has the same ordinate axis as Figure 2 and 8. Figure 9 was drawn by combining three figures (Figure 2, 6, and 8) to calculate the reinforcing steel and resistance of the columns, considering the second-order effect directly. The general application of Figure 9 is as follows:

- Connect the origin and $l_0/d \cdot \sqrt{-n}$ to get line 1.
- From m_1, draw line 2 parallel to line 1.
- From the intersection of *n* curve and line 2, draw a horizontal line 3.
- m_2 is obtained at the ordinate of line 3.
- From the abscissa of *n*, draw a vertical line 4 to meet line 3 to get an intersection, which is ω.

In addition, the effects of concrete shrinkage and creep were ignored in the analysis to avoid numerous nomograms that would depend on concrete shrinkage and creep.

4.2. Specifical examples for the nomogram (Figure9)

To determine the quantity of reinforcing steel required in a reinforced concrete circular column with dimensions of d = 600 mm, r_s = 270 mm and l_0 = 18 m to resist the compressive force of N = -3200 kN and the first-order bending moment M_1 = 300 kN, where the second-order effect is to be considered. The design values of the concrete and reinforcing steel are: C30/37, f_{ck} = 30 N/mm², f_{cd} = 20 N/mm², B500B, f_{yk} = 500 N/mm² and f_{yd} = 435 N/mm². First order dimensionless bending moment m_1 is given by:

$$m_1 = \frac{4M_1}{f_{cd}\pi d^3} = \frac{4\times300\times10^6}{20\times\pi\times600^3} = 0.088$$

n is computed as:

$$n = \frac{4N}{\pi d^2 f_{cd}} = \frac{4\times-3200\times10^3}{\pi\times600^2\times20} = -0.566$$

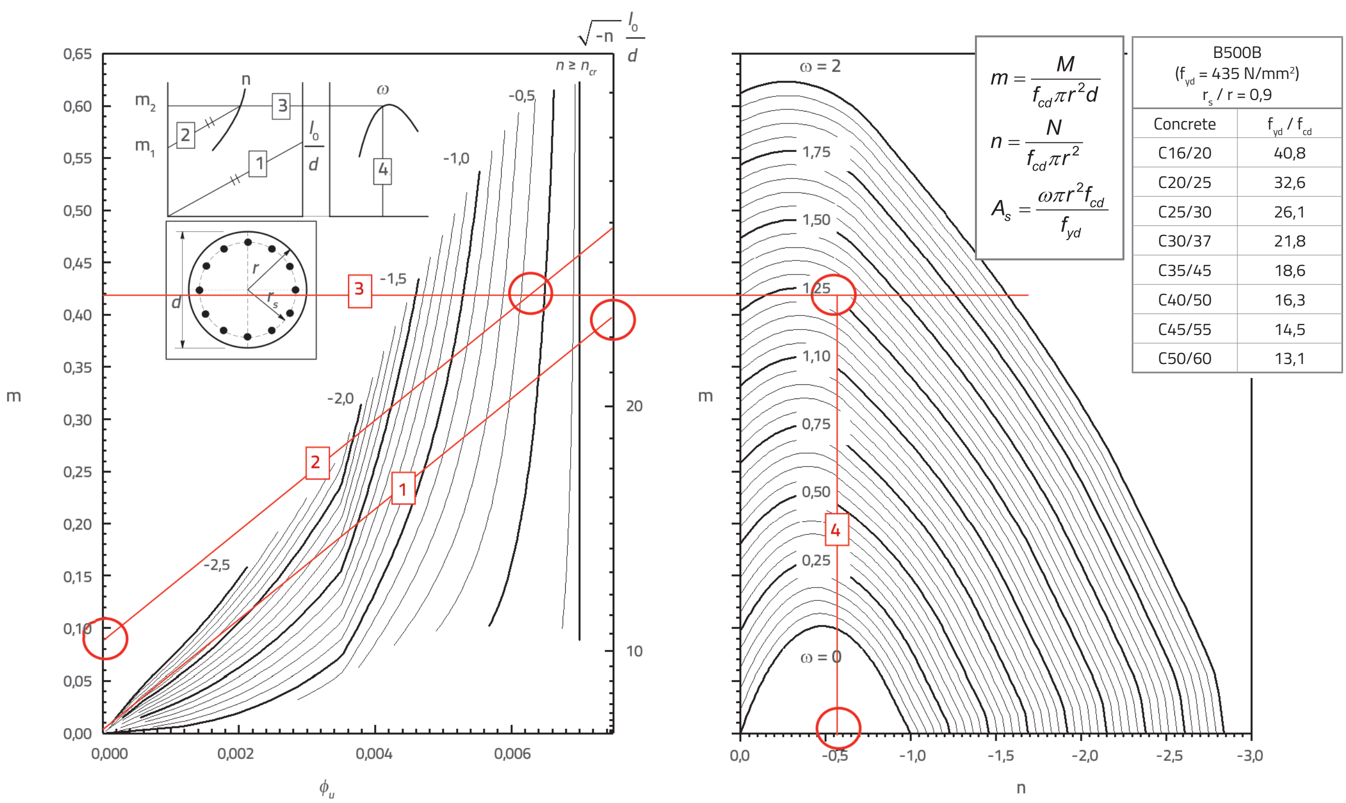

Figure 9. Reinforcing steel and resistance calculation nomogram of circular RC columns

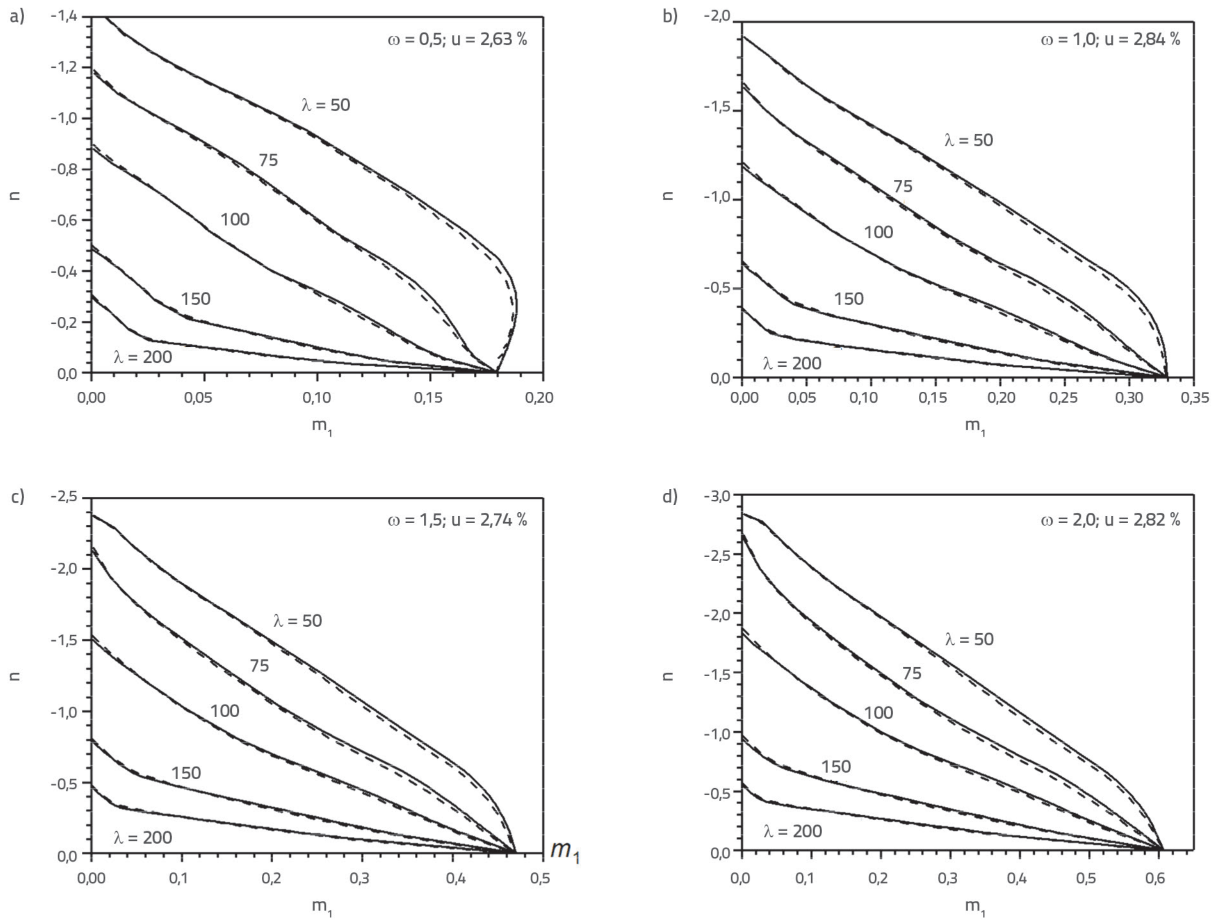

Figure 10. Comparison between the exact (dashed line) and nomogram (solid line) interaction diagrams for some RC columns with circular cross-section: r_s/r = 0,9, f_{yd} = 435 N/mm²

As shown in Figure 9, the origin and $l_0/d \cdot \sqrt{-n}$ = 22.570 are connected to obtain line 1. Based on m_1 = 0.088, line 2 parallel to line 1 can be drawn. Line 3 is obtained by drawing a horizontal line through the intersection of n = -0.566 curve and line 2. The total dimensionless bending moment m_2 = 0.419 can be found by the ordinate of line 3. Line 4 is obtained by drawing a vertical line through the intersection of line 3 and curves of the relationship between m and n. Then, ω can be obtained and the area of the reinforcing steel can be computed as:

$$A_s = \frac{\omega \pi d^2 f_{cd}}{4 f_{yd}} = \frac{1.25 \times \pi \times 600^2 \times 20}{4 \times 435} = 16174 \text{ mm}^2$$

The total bending moment is:

$$M_2 = \frac{m_2 \cdot f_{cd} \pi d^3}{4} = \frac{0.419 \times 20 \times \pi \times 600^3}{4} = 1422 \text{ kNm}$$

4.3. Examples for comparing the nomogram and exact solution

The nomogram for RC columns with circular cross sections was further validated by comparison with the exact solution. This comparison confirmed the satisfactory accuracy of the nomograms. The maximum absolute errors $\tilde{\delta}$ of the nomograms are shown in Figure 10. These results confirmed that the nomogram provided an excellent approximation of the exact method.

5. Conclusions

Based on the strain method, an analytical formula for calculating the reinforcement and bearing capacity of the circular section was derived, and the corresponding nomograms were as follows:

- (first-order) interaction diagrams of the three parameters (n–m_1–ω), from which the reinforcement ratio ω can be calculated from n and m_1.
- (second-order) interaction diagrams of the four parameters (n–m_1–l_0–ω), from which the second-order moment m_2 and corresponding mechanical reinforcement ratio ω can be calculated from n, m_1, and l_0.

The greatest advantage of the method in this study is that the nonlinear calculation problem can be solved without iteration;

that is, after providing the cross-section of the concrete and rebar, the stresses and internal forces can be computed from the strains, and all solutions in the entire ultimate strain range can be obtained.

All nomograms have a dimensionless form, which can be used for any diameter of the circular cross section and concrete strength classes of C50/60 and below, and exhibit strong practicability. The principle of this method can also be applied to high-strength concrete or fiber concrete.

Acknowledgements

The research described in This study was financially supported by the National Natural Science Foundation of China.

REFERENCES

[1] Vaz Rodrigues, R.: A new technique for ultimate limit state design of arbitrary shape RC sections under biaxial bending, Engineering Structures, 104 (2015), pp. 1-17. doi: https://doi.org/10.1016/j.engstruct.2015.09.016

[2] EN1992 EC2, Design of concrete structures, 2004.

[3] Smolcic, Z., Grandic, D.: Interaction diagrams for reinforced concrete circular cross-section, GRADEVINAR, 64 (2012) 1, pp. 23-31.

[4] Quaranta, G., Trentadue, F., Marano, G.C.: Closed-form approximation of the axial force-bending moment interaction diagram for hollow circular reinforced concrete cross-sections, Engineering Structures, 153 (2017), pp. 516-524.

[5] Babazadeh, A., Burgueno, R., Silva, P.F.: P-δ Effects on the Plastic Region of RC Bridge Columns: Closed-Form Solution, Journal of Structural Engineering, 142 (2016), pp. 1-11.

[6] De Araújo, J.M.: Comparative study of the simplified methods of Eurocode 2 for second order analysis of slender reinforced concrete columns, Journal of Building Engineering, 14 (2017), pp. 55-60. doi: https://doi.org/10.1016/j.jobe.2017.10.003

[7] Bonet, J.L., Romero, M.L., Miguel, P.F.: Effective flexural stiffness of slender reinforced concrete columns under axial forces and biaxial bending, Engineering Structures, 33 (2011) 3, pp. 881-893.

[8] Kwak, H.G., Kim, J.K.: Ultimate resisting capacity of slender RC columns, Computers & Structures, 82 (2004) 11, pp. 901-915. doi: https://doi.org/10.1016/j.compstruc.2004.02.019

[9] Kwak, H.G., Kim, J.K.: Nonlinear behavior of slender RC columns: (1). Numerical formulation, Construction and Building Materials, 20 (2006) 8, pp. 527-537. doi: https://doi.org/10.1016/j.conbuildmat.2005.01.036

[10] Kwak, H.G., Kwak, J.H.: An improved design formula for a biaxially loaded slender RC column, Engineering Structures, 32 (2010) 1, pp. 226-237, doi: https://doi.org/10.1016/j.engstruct.2009.09.009

[11] Majewski, T., Bobinski, J., Tejchman, J.: FE analysis of failure behaviour of reinforced concrete columns under eccentric compression, Engineering Structures, 30 (2008) 2, pp. 300-317. doi: https://doi.org/10.1016/j.engstruct.2007.03.024

[12] Perdomo, C., Monteiro, R.: Simplified damage models for circular section reinforced concrete bridge columns, Engineering Structures, 217 (2020), pp. 110794. doi: https://doi.org/10.1016/j.engstruct.2020.110794

[13] Porras, R., Carmona, J.R., Yu, R.C., et al.: Experimental study on the fracture of lightly reinforced concrete elements subjected to eccentric compression, Materials and Structures, 49 (2016) 1, pp. 87-100. doi: 10.1617/s11527-014-0476-3

[14] Xu, J., Gong, J.X.: Nonlinear Second-Order Effect of Non-Sway Columns, Aci Structural Journal, 110 (2013) 5, pp. 755-765.

[15] Barrera, A.C., Bonet, J.L., Romero, M.L., et al.: Ductility of slender reinforced concrete columns under monotonic flexure and constant axial load, Engineering Structures, 40 (2012), pp. 398-412. doi: https://doi.org/10.1016/j.engstruct.2012.03.012

[16] de Araujo, J.M.: A non-linear model for analysis and design of slender reinforced-concrete columns, Magazine Of Concrete Research, 71 (2019) 6, pp. 287-297. doi: 10.1680/jmacr.17.00428

[17] Kowalsky, M.J.: Deformation limit states for circular reinforced concrete bridge columns, Journal of Structural Engineering-Asce, 126 (2000) 8, pp. 869-878. doi: 10.1061/(asce)0733-9445(2000)126:8(869)

[18] Gentile, R., Porco, F., Raffaele, D., Uva, G.: Simplified moment-curvature relationship in analytical form for circular RC sections, Bulletin of the New Zealand Society for Earthquake Engineering, 51 (2018) 3, p. 14.

[19] Donghua, Z., Qiongfen, W., Jiang, F.: A new method for the calculation of reinforcement of concrete: general dimensionless designing diagram, Engineering Mechanics, 27 (2010) 01, pp. 165-172.

[20] Laora, R.D., Galasso, C., Mylonakis, G., et al.: A simple method for N-M interaction diagrams of circular reinforced concrete cross sections, Structural Concrete, 21 (2020) 1, pp. 48-55. doi: 10.1002/suco.201900139

[21] Kaicheng, Y., Donghua, Z., Chao, S., et al.: Reinforcement Calculation of Circular and Annular Cross Section, Tehnički Vjesnik, 27 (2020) 6, pp. 2008-2015. doi: 10.17559/tv-20190709112144

[22] Chao, S., Dong-hua, Z., Xu, C., et al.: Simplified calculation method for second-order bearing capacity of circular section columns, Journal of Jilin University (Engineering and Technology Edition), 50 (2020) 06, pp. 2178-2185. doi: 10.13229/j.cnki.jdxbgxb20190787

[23] Ou, Y.C., Kurniawan, R.A., Kurniawan, D.P., et al.: Plastic hinge length of circular reinforced concrete columns, Computers and Concrete, An International Journal, 10 (2012) 6, pp. 663-681.